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RESEARCH METHODS & REPORTING

STATISTICS NOTES

Brackets (parentheses) in formulas

Douglas G Altman *professor of statistics in medicine*[1], J Martin Bland *professor of health statistics*[2]

[1]Centre for Statistics in Medicine, University of Oxford, Oxford OX2 6UD; [2]Department of Health Sciences, University of York, York YO10 5DD

Each year, new health sciences postgraduate students in York are given a simple maths test. Each year the majority of them fail to calculate $20 – 3 \times 5$ correctly. According to the conventional rules of arithmetic, division and multiplication are done before addition and subtraction, so $20 – 3 \times 5 = 20 – 15 = 5$. Many students work from left to right and calculate $20 – 3 \times 5$ as $17 \times 5 = 85$. If that was what was actually meant, we would need to use brackets: $(20 – 3) \times 5 = 17 \times 5 = 85$. Brackets tell us that the enclosed part must be evaluated first. That convention is part of various mnemonic acronyms which indicate the order of operations, such as BODMAS (Brackets, Of (that is, power of), Divide, Multiply, Add, Subtract) and PEMDAS (Parentheses, Exponentiation, Multiplication, Division, Addition, Subtraction).[1]

Schoolchildren learn the basic rules about how to construct and interpret mathematical formulas.[1] The conventions exist to ensure that there is absolutely no ambiguity, as mathematics (unlike prose) has no redundancy, so any mistake may have serious consequences. Our experience is that mistakes are quite common when formulas are presented in medical journal articles. A particular concern is that brackets are often omitted or misused. The following examples are typical and we mean nothing personal by choosing them.

Example 1

In a discussion of methods for analysing diagnostic test accuracy, Collinson [2] wrote:

> Sensitivity = TP/TP + FN

where TP = true positive and FN = false negative. The formula should, of course, be:

> Sensitivity = TP/(TP + FN).

Example 2

For a non-statistical example, Leyland [3] wrote that the total optical power of the cornea is:

$$P = P_1 + P_2 – t/n_2(P_1P_2)$$

where $P_1 = n_2 – n_1/r_1$ and $P_2 = n_3 – n_2/r_2$. Here n_1, n_2, and n_3 are refractive indices, r_1, r_2, and t are distances in metres, and P, P_1, and P_2, are powers in dioptres. But he should have written $P_1 = (n_2 – n_1)/r_1$ and $P_2 = (n_3 – n_2)/r_2$. $P_1 = n_2 – n_1/r_1$ is clearly wrong dimensionally, as P_1 is dioptres, 1/metre, n_2 and n_1 are ratios and so pure numbers, and r_1 is in metres. Also, it is not clear whether $t/n_2(P_1P_2)$ means $(t/n_2)\,P_1P_2$, which it does, or $t/(n_2P_1P_2)$.

Do such errors matter? Certainly. In our experience the calculations are usually correct in the paper, but anyone using the published formula would go wrong. Sometimes, however, the incorrect formula was used, as in the following case.

Example 3

In their otherwise exemplary evaluation of the chronic ankle instability scale, Eechaute et al[4] made a mistake in their formula for the minimal detectable change (MDC) or repeatability coefficient,[5] writing: $MDC = 2.04 \times \sqrt{(2 \times SEM)}$. Here SEM is the standard error of a measurement or within subject standard deviation.[5] This formula uses 2.04 where 2 or 1.96 is more usual,[6] but, much more seriously, the SEM should not be included within the square root, as the brackets indicate. This might be dismissed as a simple typographical error, but the authors actually used this incorrect formula. Their value of SEM was 2.7 points, so they calculated the minimal detectable change as $2.04 \times \sqrt{(2 \times 2.7)} = 4.7$. They should have calculated $2.04 \times \sqrt{(2)} \times 2.7 = 7.8$. Their erroneous formula makes the scale appear considerably more reliable than it actually is.[6] The formula is also wrong in terms of dimensions, because the minimum clinical difference should be in the same units as the measurement, not in square root units.

Some mistakes in formulas may be present in a submitted manuscript, but others might be introduced in the publication process. For example, problems sometimes arise when a displayed formula is converted to an “in-text” formula as part of the editing, and the implications are not realised or not noticed by either editing staff or authors. Often it is necessary to insert brackets when reformatting a formula. So the simple formula:

Correspondence to: DG Altman doug.altman@csm.ox.ac.uk

$$\frac{p}{1-p}$$

should be changed to $p/(1 - p)$ if moved to the text.

Formulas in published articles may be used by others, so mistakes may lead to substantive errors in research. It is essential that authors and editors check all formulas carefully.

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