

RESEARCH METHODS & REPORTING

STATISTICS NOTES

How to obtain the P value from a confidence interval

Douglas G Altman *professor of statistics in medicine*¹, J Martin Bland *professor of health statistics*²

¹Centre for Statistics in Medicine, University of Oxford, Oxford OX2 6UD; ²Department of Health Sciences, University of York, Heslington, York YO10 5DD

We have shown in a previous *Statistics Note*¹ how we can calculate a confidence interval (CI) from a P value. Some published articles report confidence intervals, but do not give corresponding P values. Here we show how a confidence interval can be used to calculate a P value, should this be required. This might also be useful when the P value is given only imprecisely (eg, as $P < 0.05$). Wherever they can be calculated, we are advocates of confidence intervals as much more useful than P values, but we like to be helpful.

The method is outlined in the box below in which we have distinguished two cases.

(a) P from CI for a difference (no transformation needed)

The simple case is when we have a CI for the difference between two means or two proportions. For example, participants in a trial received antihypertensive treatment with or without pravastatin. The authors report that pravastatin performed slightly worse than a placebo. The estimated difference between group means was 1.9 (95% CI -0.6 to 4.3) mm Hg.⁴ What was the P value?

Following the steps in the box above we calculate P as follows:

$$SE = [4.3 - (-0.6)] / (2 \times 1.96) = 1.25;$$

$$z = 1.9 / 1.25 = 1.52;$$

$$P = \exp(-0.717 \times 1.52 - 0.416 \times 1.52^2) = 0.13.$$

In this paper, the authors did indeed publish a P value of 0.13,⁴ as we have estimated from their confidence interval.

(b) CI for a ratio (log transformation needed)

The calculation is trickier for ratio measures, such as risk ratio, odds ratio, and hazard ratio. We need to log transform the estimate and confidence limits, so that *Est*, *l*, and *u* in the box are the logarithms of the published values.

For example, in a meta-analysis of several studies comparing single versus bilateral mammary artery coronary bypass grafts

Taggart et al presented a hazard ratio of 0.81; 95% CI 0.70 to 0.94.⁵ They did not quote the P value.

Following the steps in the box we calculate P as follows:

$$Est = \log(0.81) = -0.211$$

$$l = \log(0.70) = -0.357, u = \log(0.94) = -0.062$$

$$SE = [-0.062 - (-0.357)] / (2 \times 1.96) = 0.0753.$$

$$z = -0.211 / 0.0753 = -2.802. \text{ We take the positive value of } z, 2.802.$$

$$P = \exp(-0.717 \times 2.802 - 0.416 \times 2.802^2) = 0.005.$$

Limitations of the method

The formula for P is unreliable for very small P values and if your P value is smaller than 0.0001, just report it as $P < 0.0001$.

The methods described can be applied in a wide range of settings, including the results from meta-analysis and regression analyses. The main context where they are not correct is small samples where the outcome is continuous and the analysis has been done by a *t* test or analysis of variance, or the outcome is dichotomous and an exact method has been used for the confidence interval. However, even here the methods will be approximately correct in larger studies with, say, 60 patients or more.

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1 Altman DG, Bland JM. How to obtain a confidence interval from a P value. *BMJ* 2011;342:d2090.

2 Lin J-T. Approximating the normal tail probability and its inverse for use on a pocket calculator. *Appl Stat* 1989;38:69-70.

Steps to obtain the P value from the CI for an estimate of effect (*Est*)*(a) P from CI for a difference*

If the upper and lower limits of a 95% CI are *u* and *l* respectively:

1 calculate the standard error: $SE = (u - l)/(2 \times 1.96)$

2 calculate the test statistic: $z = Est/SE$

3 calculate the P value²: $P = \exp(-0.717 \times z - 0.416 \times z^2)$.

(b) P from CI for a ratio

For a ratio measure, such as a risk ratio, the above formulas should be used with the estimate *Est* and the confidence limits on the log scale (eg, the log risk ratio and its CI).

Notes

All P values are two sided.

All logarithms are natural (ie, to base *e*).³

"exp" is the exponential function.

The formula for P works only for positive *z*, so if *z* is negative we remove the minus sign.

For a 90% CI, we replace 1.96 by 1.65; for a 99% CI we use 2.57.

- 3 Mancia G, Parati G, Revera M, Bilo G, Giuliano A, Veglia F, et al. Statins, antihypertensive treatment, and blood pressure control in clinic and over 24 hours: evidence from PHYLLIS randomised double blind trial. *BMJ* 2010;340:c1197.
- 4 Taggart DP, D'Amico R, Altman DG. Effect of arterial revascularisation on survival: a systematic review of studies comparing bilateral and single internal mammary arteries. *Lancet* 2001;358:870-5.

- 5 Bland JM, Altman DG. Logarithms. *BMJ* 1996;312:700.

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