The Nigerian edition of the *BMJ* will not claim to have all the answers to this multitude of problems. Rather, the mission statement of the publication is to stimulate essential debate between patients and doctors, between doctors and planners of healthcare delivery, and within the the medical and other health professions in the country with a view to facing realities and recognising that a healthy population produces a successful economy. Hard choices have to be made, but Nigeria cannot continue to treat the health of the nation as a residual matter. To doctors inside Nigeria, the Nigerian edition of the *BMJ* will bring access to first rate, peer reviewed research papers which otherwise most would not be able to afford. It will serve the doctors so that they can serve their patients well.

1 Federal Ministry of Health, Department of Planning and Statistics. *Nigeria health profile (1992-1993)*. Lagos: Federal Ministry of Health, 1993:1-6,54.
2 Nigerian Medical Association. *National medical directory*. Lagos: Pacific Printers, 1993. (PP75-81.)

Statistics Notes

Measurement error proportional to the mean

J Martin Bland, Douglas G Altman

This is the 23rd in a series of occasional notes on medical statistics

Department of Public Health Sciences, St George's Hospital Medical School, London SW17 0RE
J Martin Bland, *professor of medical statistics*

ICRF Medical Statistics Group, Centre for Statistics in Medicine, Institute of Health Sciences, PO Box 777, Oxford OX3 7LF
Douglas G Altman, *head*

Correspondence to:
Professor Bland.

BMJ 1996;313:106

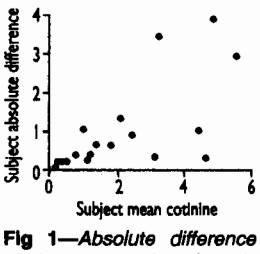

Fig 1—*Absolute difference against mean for data in table 1*

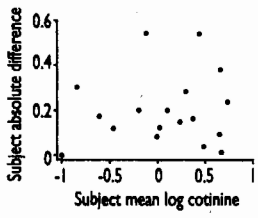

Fig 2—*Absolute difference against mean after $\log_{10}$ transformation*

We often need to know the error with which measurements are made—for example, so that we can decide whether the change in a clinical observation represents a real change in a patient's condition. We have discussed previously the within-subject standard deviation as a practical index of measurement error.[1] We said that this approach should be used when the measurement error was not related to the magnitude of the measurement and recommended that we plot the subject standard deviation against the subject mean to check this. Table 1 shows some duplicate salivary cotinine measurements taken from a larger study. Figure 1 shows absolute subject difference against subject mean, which is equivalent to a standard deviation versus mean plot when we have only two measurements per subject.[1] If we are to use the within-subject standard deviation as an index of measurement error we need the subject standard deviation to be independent of the subject mean. Here, there is a clear relationship, the variability increasing with the magnitude. We can test this using a rank correlation coefficient if we wish; here Kendall's $\tau = 0.62$, $P = 0.0001$. Under these circumstances a logarithmic transformation of the data almost always solves the problem, but we can check by plotting log standard deviation against log mean. For these data the slope is 0.9; as this is very close to 1 the subject standard deviation is roughly proportional to the subject mean and a log transformation is indicated.[2] Figure 2 shows the plot of absolute difference versus subject mean for the log transformed data. There is now no evidence of a relationship (Kendall's $\tau = 0.07$, $P = 0.7$).

As the variability is now independent of the magnitude of the measurement, we can calculate the within-subject standard deviation[1] as $\sigma_w = 0.175$. This is a standard deviation on the logarithmic scale, so we need to antilog it before we can interpret it easily. We will denote the antilog of σ_w *by* $a_{\sigma w}$. For the cotinine data, $a_{\sigma w} = 1.496$.

When we antilog σ_w we have a ratio, not a quantity measured in the units of the original data. This is because to calculate a standard deviation we subtract the mean from each observation. Subtracting on the logarithmic scale is equivalent to dividing on the natural scale.[3] Dividing the observation by the mean in this way produces a dimensionless ratio. Hence $a_{\sigma w}$ is not a standard deviation in the original units, but a related quantity sometimes referred to as the *geometric standard deviation.*

To estimate one standard deviation on either side of the observed value, we should multiply and divide by $a_{\sigma w}$. The difference between a subject's measurement and the true value would be expected to be less than $1.96\sigma_w$ for 95% of observations.[1] To get the equivalent of 1.96 log scale standard deviations on either side of an observed value we would multiply and divide on the natural scale by $a_{\sigma w}^{1.96}$ or approximately $a_{\sigma w}^{2}$. This procedure gives limits which should include the subject's mean for 95% of observations. Thus for the cotinine data we would divide and multiply by $1.496^2 = 2.238$. A measurement of 2 ng/ml would tell us that the person's true value probably lies somewhere between $2/2.238 = 0.9$ and $2 \times 2.238 = 4.5$ ng/ml.

Table 1—*Duplicate salivary cotinine measurements for a group of Scottish schoolchildren (ng/ml) (D Strachan, personal communication)*

Subject No	Measurement 1st	Measurement 2nd	Subject No	Measurement 1st	Measurement 2nd
1	0.1	0.1	11	1.2	0.9
2	0.2	0.1	12	1.9	2.8
3	0.2	0.3	13	2.0	1.4
4	0.3	0.4	14	2.7	1.4
5	0.3	0.4	15	2.8	6.8
6	0.4	0.3	16	3.2	2.9
7	0.4	1.4	17	4.7	4.5
8	0.8	0.5	18	4.9	1.4
9	1.0	1.6	19	4.9	3.9
10	1.1	0.9	20	7.0	4.0

Multiplying on the natural scale is equivalent to adding on the log scale. Multiplying a subject's actual measurement by $a_{\sigma w}$ is equivalent to adding one standard deviation on the log scale. Provided the standard deviation is not large compared with the level of the measurement, $a_{\sigma w}-1$ is approximately equal to the standard deviation expressed as a proportion of the measurement. The ratio of standard deviation to mean is called a *coefficient of variation,* and here $a_{\sigma w}-1$ is the within-subject coefficient of variation.

For the cotinine data the estimated coefficient of variation is $1.496-1 = 0.496$ or 49.6%. This is rather too large for the approximation to be reliable.

The within-subject variability for salivary cotinine seems very large, but the possible range of values, from these very lightly exposed children to heavy smokers, is very wide and salivary cotinine is sufficiently precise to distinguish between many different levels of exposure. The precision of a method of measurement must be interpreted in the light of its intended use.

1 Bland JM, Altman DG. Measurement error and correlation coefficients. *BMJ* 1996;313:41-2.
2 Bland JM, Altman DG. Transforming data. *BMJ* 1996;312:770.
3 Bland JM, Altman DG. Logarithms. *BMJ* 1996;312:700.