



RESEARCH METHODS & REPORTING

Statistics Notes: What is a percentage difference?

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We use percentages to express differences as a fraction of the whole. Suppose we want to know the percentage difference in mean height of British adults aged 20 years, 177.3 cm for men and 163.6 cm for women, a difference of 13.7 cm.¹ Women are $100 \times (13.7/177.3) = 7.7\%$ shorter than men, whereas men are $100 \times (13.7/163.6) = 8.4\%$ taller than women. There are two percentage differences, depending on which sex is used as the divisor—this can be confusing. The same problem does not arise with the absolute difference—women are 13.7 cm shorter than men, and men are 13.7 cm taller than women.

Often one of the two numbers to be compared is obviously appropriate as the divisor, such as the first of two measurements taken some time apart; the percentage difference is then the percentage change over time. But often neither measurement is an obvious baseline, and neither of the two percentages is satisfactory. What is the percentage difference in height between the two sexes: is it 7.7% or 8.4%? The answer could be neither, either, and both.

When the difference is small the two percentages are very similar. At 11 years of age, girls are on average 0.523% taller than boys, and boys are 0.520% shorter than girls. The two percentage differences diverge as the two numbers become more different. The married couple recorded as being most dissimilar in height² were 188 cm and 94 cm tall, a difference of 94 cm. He was 100% taller than her, while she was 50% shorter than him.

This is clearly an extreme example, but it highlights the potential for confusion. A way forward is to define an alternative form of percentage difference with the mean of the two numbers as divisor:

Percentage difference = $100 \times (\text{difference} / \text{mean})$.

So, for the vertically challenged couple, whose mean is $(188+94)/2 = 141$ cm, the percentage difference would be

$100 \times (94/141) = 66.7\%$. This lies between the two conventional percentage differences and is unchanged if the two heights are swapped, +66.7% or -66.7%. It is a *symmetric* percentage difference, which matches the symmetry of the absolute difference in height, +94 cm or -94 cm. Note, though, that its value depends on which form of mean is used—using the geometric or harmonic mean instead of the arithmetic mean would give a different answer.

However, there is a second problem with the conventional percentage difference—it does not add up. Take a preterm infant whose weight increases by 10% each week for two weeks. You might expect her weight to have increased overall by $2 \times 10\% = 20\%$, but you would be wrong: the true figure is 21%. And similarly, if her weight were to rise by 10% one week and fall by 10% the next, the two do not cancel out: her weight at the end would be only 99% of her starting weight.

These examples show that, as well as not being symmetric, the percentage difference is not *additive*. And unlike symmetry, additivity is not always achieved by using the mean as divisor.

Percentage differences arise in other contexts, such as fractional standard deviations and fractional regression coefficients. A separate Statistics Note³ shows how the concepts are linked and how to calculate a percentage difference that is both symmetric and additive.

1 Freeman JV, Cole TJ, Chinn S, Jones PRM, White EM, Preece MA. Cross sectional stature and weight reference curves for the UK, 1990. *Arch Dis Child* 1995;73:17-24. 10.1136/adc.73.1.17 7639543

2 *The Guinness Book of Records*: 2017. Guinness World Records, 2016.

3 Cole TJ, Altman DG. Statistics Notes: Percentage differences, symmetry, and natural logarithms. *BMJ* 2017;358:j3663. 10.1136/bmj.j3663.

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