Statistics at Square One

XVI—The χ^2 tests (concluded)

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Fit of class to sample

In Part XIV we compared two samples by the χ^2 test to answer the question: Are the distributions of the members of these two samples between five classes significantly different? Another way of putting this is to ask, Does each sample fit the classes in the same sort of way? A converse approach that is sometimes useful is to ask, Does each class fit the sample in the same sort of way?

For example, Dr Scarlet is an industrial medical officer of a large factory whose employees want to be immunised against influenza. Five vaccines of various types based on the current viruses are available, but nobody knows which is preferable to another. Dr Scarlet finds that 1350 employees agree to be immunised with one of the vaccines in the first week of December, so he divides the total up into five approximately equal groups. Disparities occur between their total numbers owing to the layout of the factory complex. In the first week of the following March he examines the records he has been keeping to see how many employees got influenza and how many did not. These records are classified by the type of vaccine (table 16.1).

TABLE 16.1—*People who did or did not get influenza after inoculation with one of five vaccines*

Type of vaccine	Numbers of employees		
	Got influenza	Avoided influenza	Total
I	43	237	280
II	52	198	250
III	25	245	270
IV	48	212	260
V	57	233	290
Total	225	1125	1350

In table 16.2 the figures are analysed by the χ^2 test. For this we have to determine what are the expected values. Dr Scarlet's null hypothesis is that there is no difference between the vaccines in their efficacy against influenza. We therefore assume the proportion of employees contracting influenza is the same for each vaccine as it is for all combined. This proportion is derived from the total who got influenza, and is 225/1350. To find the expected number in each vaccine group who contracted the disease we multiply the actual numbers in the "Total" column of table 16.1 by this proportion. Thus $280 \times (225 \div 1350) = 46{\cdot}7$; $250 \times (225 \div 1350) = 41{\cdot}7$; and so on. Likewise the proportion who did not get influenza is 1125/1350. Again the expected numbers are calculated in the same way from the totals in table 16.1, so that $280 \times (1125 \div 1350) = 233{\cdot}3$; $250 \times (1125 \div 1350) = 208{\cdot}3$; and so on. The procedure is thus the same as described in Part XIV and tables 14.1 and 14.2.

TABLE 16.2—*Calculation of χ^2 on figures in table 16.1*

Type of vaccine	Expected numbers		O–E		$(O-E)^2/E$	
	Got influenza	Avoided influenza	Got influenza	Avoided influenza	Got influenza	Avoided influenza
I	46·7	233·3	−3·7	3·7	0·293	0·059
II	41·7	208·3	10·3	−10·3	2·544	0·509
III	45·0	225·0	−20·0	20·0	8·889	1·778
IV	43·3	216·7	4·7	−4·7	0·510	0·102
V	48·3	241·7	8·7	−8·7	1·567	0·313
Total	225·0	1125·0	0	0	13·803	2·761

$\chi^2 = 16{\cdot}564$. DF = 4. $0{\cdot}01 > P > 0{\cdot}001$.

The calculations in table 16.2 show that χ^2 with 4 degrees of freedom is 16·564, and $0{\cdot}01 > P > 0{\cdot}001$. This is a highly significant result. But what does it mean?

Inspection of table 16.2 shows that much the largest contribution to the total χ^2 comes from the figures for vaccine III. They are 8·889 and 1·778, which together equal 10·667. If this figure is subtracted from the total χ^2, $16{\cdot}564 - 10{\cdot}667 = 5{\cdot}897$. This gives an approximate figure for χ^2 for the remainder of the table with 3 degrees of freedom (by removing the vaccine III now we have reduced the table to 4 rows and 2 columns). We then find that $0{\cdot}5 > P > 0{\cdot}1$, a non-significant result. But this is only a rough approximation. To check it exactly we apply the χ^2 test to the figures in table 16.1 minus the row giving those for vaccine III. In other words, the test is now performed on the figures for vaccines I, II, IV, and V. On these figures $\chi^2 = 2{\cdot}983$, DF = 3, $0{\cdot}5 > P > 0{\cdot}1$. Thus the probability falls within the same broad limits as by the approximate short cut given above. We can conclude that the figures for vaccine III are responsible for the highly significant result of the total χ^2 of 16·564.

But this is not quite the end of the story. Before concluding from these figures that vaccine III is superior to the others we ought to carry out a check on other possible explanations for the disparity. The process of randomisation in the choice of the persons to receive each of the vaccines should on the average have balanced out any differences between the groups, but some may have remained by chance. The sort of questions worth examining now are: Were the people receiving vaccine III as likely to be exposed to infection as those receiving the other vaccines? Could they have had a higher level of immunity from previous infection? Were they of comparable socioeconomic status? Of similar age on average? Were the sexes comparably distributed? Though these characteristics should have been more or less equalised when the groups were chosen in the first place, it is as well to check that they have in fact been equalised before attributing the numerical discrepancy in the results to the potency of the vaccine.

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Theoretical distribution

In the cases so far discussed the observed values in one sample have been compared with the observed values in another. But sometimes we want to compare the observed values in one sample with a theoretical distribution.

For example, Dr Pink was studying 18 cases of the congenital disease, Everley's syndrome (Part XI). He found that these 18 people together with all their brothers and sisters made a total sibship of 64 all living. It is possible to identify the carrier state in this disease, and among the 46 sibs who did not have the disease he found 38 carriers. According to the genetic theory for the inheritance of Everley's syndrome a sibship of this kind would contain one-quarter with the disease, one-half as carriers, and one-quarter normal. Do Dr Pink's cases depart from those proportions?

The data are set out in table 16.3. The expected numbers are calculated by applying the theoretical proportions to Dr Pink's total, namely $0{\cdot}25 \times 64$, $0{\cdot}5 \times 64$, and $0{\cdot}25 \times 64$. Thereafter the procedure is the same as in previous calculations of χ^2. In this case it comes to 5·375. The χ^2 table is entered at 2 degrees of freedom. The value of $P > 0{\cdot}05$ does not reach a significant level. Consequently, the null hypothesis of no difference between the observed distribution and the theoretically expected one is *not* disproved. Dr Pink's data do not depart significantly from the expected frequencies.

TABLE 16.3—*Calculation of χ^2 from comparison between actual distribution and theoretical distribution*

	Observed cases	Theoretical proportions	Expected cases	O–E	(O–E)²/E
Diseased	18	0·25	16	2	0·25
Carriers	38	0·5	32	6	1·125
Normal	8	0·25	16	−8	4·0
Total	64	1·0	64	0	5·375

$\chi^2 = 5{\cdot}375$. DF = 2. $0{\cdot}10 > P > 0{\cdot}05$.

Exercise 16. The construction of a reservoir several years ago in an African country brought bilharzia to four villages that stand near it. Measures taken by special teams from each village to eliminate the snails were only partially effective, and a survey in these villages gave the following figures for residual cases of bilharzia (with village population in parentheses): village A, 14 (103); village B, 11 (92); village C, 39 (166); village D, 31 (221). What are the χ^2 and P values for the distribution of the cases in these villages? Do they suggest that any one village has significantly more cases than the others? *Answer*: $\chi^2 = 8{\cdot}949$, DF = 3, $0{\cdot}05 > P > 0{\cdot}02$. Yes, village C; if this is omitted the remaining villages give $\chi^2 = 0{\cdot}241$, DF = 2, $P > 0{\cdot}5$. (Both χ^2 tests by quick method).

Contemporary Themes

Career problems of women doctors

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Summary

Information was received from 61 women doctors who were having difficulty continuing with medical careers. Two main problems were disclosed. Firstly, despite the special arrangements made for women doctors, it is difficult to obtain postgraduate training. The provision of supernumerary posts does not seem to offer a satisfactory solution. Secondly, doctors who have completed postgraduate training but cannot yet return to full-time work are unable to obtain posts at an appropriate level. Both of these problems stem primarily from the need for part-time work by the mothers of young children. Most of the doctors wish to return to full-time or nearly full-time work when family responsibilities are fewer. In view of the increasing proportion of women doctors it seems important that large numbers are not unnecessarily lost from professional work. Some possible approaches to solving the problems are suggested.

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Introduction

"Women are not equal to men—but then neither are men equal to women"—ISLAMIC SAYING.

There are at present about 19 000 women doctors in Great Britain—that is, about 22% of doctors are women.[1] Thirty-five percent of the students starting medical training in 1975 were women, and this proportion may well rise in the future. Surveys of the pattern of women doctors' work indicate that at any time about half work full-time, 30-35% work part-time, and 15-20% do not work.[2-4]

With the current medical manpower shortage[5] there can be no doubt that women doctors will be needed to make an important contribution to medical services if these are to be maintained, even at present levels. Nor need it be doubted that most women doctors want to justify their choice of medical studies by working after qualification. Yet the changes in social attitudes and expectations that have made it increasingly possible for women to enter the course of study of their choice have paradoxically made it more difficult for those women with family responsibilities to free themselves for professional work by employing others to take on the domestic and child-caring tasks. This is the result partly of wider employment openings for women who once