This low number was a surprise and supported the view that its existence encouraged domiciliary care, but in the absence of information about the total work load this conclusion can only be tentative. Another reason for the lower rate of self-referral in our study may be the different nature of medical practice and public attitudes in a large city and a smaller community.

This investigation supports the contention of Patel[2] that the hospital plays a major part in primary emergency care. In these circumstances the public might benefit from an entirely hospital-based emergency medical service in urban areas. The ETS would then be unnecessary and the ambulance service and police relieved of much responsibility. An experienced doctor would receive all emergency calls (replacing the 999 call) from the public after 5 pm and at weekends. His staff would consist of hospital registrars and local general practitioners serving in rotation. On his assessment either an ambulance would be dispatched with or without a doctor in attendance or a general practitioner would make a home call. This system would ensure prompt attention in an emergency and at the same time prevent unnecessary admissions. It would also be an interesting experiment in hospital-general practice integration; a similar system has operated in The Hague since the second world war.[3]

I thank the junior staff members and medical students who assisted in this survey—in particular, Drs J Drury, A Lochrie, and I Fogelman and Mr J Gooden.

References

1 *British Medical Journal*, 1973, **3**, 248.
2 Patel, A R, *British Medical Journal*, 1971, **1**, 281.
3 *British Medical Journal*, 1976, **1**, 732.

Statistics at Square One

XV—The χ^2 tests (continued)

T D V SWINSCOW

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Fourfold tables

A special form of the χ^2 test is particularly common in practice and quick to calculate. It is applicable when the results of an investigation can be set out in a so-called "fourfold table" or "2×2 contingency table."

For example, Dr White, who had been inquiring into the blood pressures of the printers and sheep farmers in her general practice (Part VIII), believed that their wives should be encouraged to breast-feed their babies. She has records for her practice going back over 10 years in which she has noted whether the mother breast-fed the baby for at least three months or not, and these records show whether the husband was a printer or sheep farmer (or some other occupation less well represented in her practice). The figures from her records are set out in table 15.1. The disparity seems considerable, for, while 28% of the printers' wives breast-fed their babies for three months or more, as many as 45% of the farmers' wives did so. What is its significance?

Again the null hypothesis is set up that there is no difference between printers' wives and farmers' wives in the period for which they breast-fed their babies. The χ^2 test on a fourfold table may be carried out by a formula that provides a short-cut to the conclusion. If a, b, c, and d are the numbers in the cells of the fourfold table as shown,

			Total
	a c	b d	$a+b$ $c+d$
Total	$a+c$	$b+d$	$a+b+c+d$

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χ^2 is calculated from the following formula:

$$\frac{(a\,d-b\,c)^2\,(a+b+c+d)}{(a+b)\,(c+d)\,(b+d)\,(a+c)}.$$

With a fourfold table there is 1 degree of freedom in accordance with the rule given last week, (number of columns minus 1)× (number of rows minus 1).

Since many electronic calculators have a capacity limited to eight digits, it is advisable not to do all the multiplication or all the division in one series of operations, lest the number become too big for the display. A suitable method is as follows:

Multiply a by d and store in memory

Multiply b by c and subtract from memory

Extract difference from memory to display .. .. $a\,d-b\,c$

Square the difference .. .. .. $(a\,d-b\,c)^2$

Divide by $a+b$ .. .. .. $\dfrac{(a\,d-b\,c)^2}{a+b}$

Divide by $c+d$ .. .. .. $\dfrac{(a\,d-b\,c)^2}{(a+b)\,(c+d)}$

Multiply by $a+b+c+d$ .. .. $\dfrac{(a\,d-b\,c)^2\,(a+b+c+d)}{(a+b)\,(c+d)}$

Divide by $b+d$ .. .. .. $\dfrac{(a\,d-b\,c)^2\,(a+b+c+d)}{(a+b)\,(c+d)\,(b+d)}$

Divide by $a+c$ .. .. .. $\dfrac{(a\,d-b\,c)^2\,(a+b+c+d)}{(a+b)\,(c+d)\,(b+d)\,(a+c)}$

With Dr White's figures we have

$$\frac{\{(36\times25)-(30\times14)\}^2\times105}{66\times39\times55\times50}=3{\cdot}418.$$

Entering the χ^2 table with 1 degree of freedom we read along the row and find that 3·418 lies between 2·706 and 3·841.

Therefore $0{\cdot}1 > P > 0{\cdot}05$. So, despite an apparently considerable difference between the printers' wives and the farmers' wives breast-feeding their babies for three months or more, the probability of its occurring by chance is more than 5%.

It should be emphasised again that the χ^2 test is done on the actual numbers of cases, not on, for example, percentages. But suppose the percentages are tested: do we get the same result?

For example, 28% of printers' wives and 45% of farmers' wives breast-fed their babies for three months or more. The difference is 17%. What is the standard error of this difference? It is calculated by the method set out in Part IX. With Dr White's figures we have

$$\text{SE diff } \% = \sqrt{\frac{28\times72}{50}+\frac{45\times55}{55}} = 9{\cdot}24.$$

The difference divided by its standard error is $17/9{\cdot}24 = 1{\cdot}84$. This just falls short of the 1·96 standard errors at the 5% level of probability. Reference to table 7.1 shows that it lies between 1·645 and 1·96, corresponding to $0{\cdot}1 > P > 0{\cdot}05$, the same as with the χ^2 test.

Small numbers

Experts differ somewhat on how small the numbers in contingency tables may be for a χ^2 test to yield an acceptable result. The following recommendations by Cochran[1] may be regarded as a sound guide. In fourfold tables a χ^2 test is inappropriate if the total of the table is less than 20, or if the total lies between 20 and 40 and the smallest expected (not observed) value is less than 5; in contingency tables with more than 1 degree of freedom it is inappropriate if more than about one-fifth of the cells have expected values less than 5 or any cell an expected value of less than 1.

When the values in a fourfold table are fairly small a "correction for continuity" devised by Yates[2] should be applied. While there is no precise rule defining the circumstances in which to use Yates's correction, a common practice is to incorporate it into χ^2 calculations on tables with a total of under 100 or with any cell containing a value less than 10. Armitage[3] goes so far as to say that "it is probably wise practice to apply it for almost all χ^2 tests for 2×2 tables." The χ^2 test on a fourfold table is then modified as follows:

$$\frac{\{(|a\,d-b\,c|)-\tfrac{1}{2}(a+b+c+d)\}^2\ (a+b+c+d)}{(a+b)\ (c+d)\ (b+d)\ (a+c)}.$$

The vertical bars on either side of $a\,d-b\,c$ mean that the smaller of those two products is taken from the larger. Half the total of the four values is then subtracted from that difference to provide Yates's correction. The effect of the correction is always to reduce the value of χ^2.

Applying it to the figures in table 15.1 gives the following result:

$$\frac{\{(36\times25)-(30\times14)-(105\div2)\}^2\times105}{66\times39\times55\times50} = 2{\cdot}711.$$

In this case $\chi^2 = 2{\cdot}711$ falls within the same range of P values as the $\chi^2 = 3{\cdot}418$ we got without Yates's correction, $0{\cdot}1 > P > 0{\cdot}05$, but the P value is closer to 0·1 than it was in the previous calculation. In fourfold tables containing lower frequencies than table 15.1 the reduction in P value by Yates's correction may be of considerable significance.

References

1 Cochran, W G, *Biometrics*, 1954, **10**, 417.
2 Yates, F, *Journal of the Royal Statistical Society*, Supplement, 1934, **1**, 217.
3 Armitage, P, *Statistical Methods in Medical Research*. Oxford, Blackwell Scientific Publications, 1971.

TABLE 15.1—*Numbers of wives of printers and farmers who breast-fed their babies for less than three months or for three months or more*

	Breast-fed for		Total
	Up to 3 months	3 months or more	
Printers' wives	36	14	50
Farmers' wives	30	25	55
Total	66	39	105

$\chi^2 = 3{\cdot}418$. DF = 1. $0{\cdot}1 > P > 0{\cdot}05$.

Exercise 15. An outbreak of pediculosis capitis is being investigated in a girls' school containing 291 pupils. Of 130 children who live in a nearby housing estate 18 were infested and of 161 who live elsewhere 37 were infested. What is the χ^2 value of the difference, and what is its significance? *Answer*: $\chi^2 = 3{\cdot}916$; $0{\cdot}05 > P > 0{\cdot}02$.

The 55 affected girls were divided into two groups of 29 and 26. The first group received a standard local application and the second group a new local application. The efficacy of each was measured by clearance of the infestation after one application. By this measure the standard application failed in 10 cases and the new application in 5. What is the χ^2 value of the difference (with Yates's correction), and what is its significance? *Answer*: $\chi^2 = 0{\cdot}931$; $0{\cdot}50 > P > 0{\cdot}10$.

Two women patients have swollen lips due, apparently, to swelling of the mucous membrane, which has a speckled appearance almost like a pompholyx. No treatment tried has had any effect. What is a possible diagnosis and treatment?

I cannot be sure of the diagnosis. Is the speckled appearance due to intraepidermal vesiculation (like pompholyx) or could it be due to prominent sebaceous glands (Fordyce spots) or the white streaks of lichen planus? If acute eczematous changes (intraepidermal vesiculation) are present then contact causes should be sought. Lipsticks, toothpastes, miscellaneous sucked objects, and even foods (for example, oranges and artichokes) could be the cause. Patch testing would help in detecting these. Treatment must be directed at removing the offending cause. Glandular cheilitis is a rare condition which might fit the description. It is due to heterotopic salivary glands. Fluid can be massaged out of the little bubbles. Lymphangioma circumscriptum should also be considered, but it is unlikely that one practitioner will see two patients with this. The deep vesicles are characteristic and resemble frog spawn. Treatment for both these conditions can only be surgical. Persistent swelling of a lip, without an obvious infective cause, should also raise the possibility of granulomatous cheilitis and sarcoidosis. If a dermatologist could see the patients and recognise the swelling unnecessary investigations might be prevented.

Is there any danger to health in the fruit of plants recently treated with systemic insecticide—for example, broad beans for blackfly or gooseberries for caterpillar?

Before any insecticide (or herbicide) is introduced, official agreement is reached about its toxicity and the conditions for which it should be used.[1] These conditions and the method of use are included in the manufacturer's literature and, provided they are followed, there is no hazard in eating fruit or vegetables which have been sprayed with these compounds. Although these compounds accumulate within the body with time, current ones do not cause any harm. Some of them are highly toxic, however, when inhaled or ingested directly. Practitioners should then contact the Poisons Information Service for advice.

1 Department of Health and Social Security, *Poisonous Chemicals Used on Farms and Gardens—notes for the guidance of medical practitioners*. London, HMSO, 1969.